MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2015/2016

PCM0235 - CALCULUS

(Foundation in Information Technology/ Foundation in Life Sciences)

7 OCTOBER 2015
9.00 a.m – 11.00 a.m
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This Question paper consists of 2 pages excluding the cover page and appendix.

2. Answer **FIVE** out of **SIX** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Write all your answers in the Answer Booklet provided.

Answer any five questions.

Question 1 (10 marks)

a) Evaluate the following limits.

i. $\lim_{x\to 0}\left(2\cos x+\frac{\sin 2x}{x}\right)$ (2 marks)

ii. $\lim_{x\to 1}\frac{\sqrt{x}-1}{x^2-1}$ (4 marks)

b) Given

$$f(x)=\begin{cases}-2+x & , \quad x<-1\\ x & , \quad -1\le x<3\\ (x-4)^2 & , \quad x\ge 3\end{cases}$$

i. Find $\lim_{x\to -1^+} f(x)$, $\lim_{x\to -1^-} f(x)$ and $\lim_{x\to -1} f(x)$. (2 marks)

ii. Determine if the function is continuous or not continuous at $x = 3$. (2 marks)

Question 2 (10 marks)

a) Find $\frac{dy}{dx}$ for $y=\frac{(x+4)^3}{\sqrt{2x-6}}$. Simplify the answer. (4 marks)

b) Find the equation of the tangent line to the curve $f(x)=2x^3+4x^2$ at $x=-1$. (2 marks)

c) Find $f''(x)$ for $f(x)=\cos(x^3+\pi x)$. (4 marks)

Question 3 (10 marks)

a) Suppose the length (x), width (y) and height (z) of a closed rectangular box are changing at the following rate: $\frac{dx}{dt}=-1m/\sec$, $\frac{dy}{dt}=2m/\sec$, $\frac{dz}{dt}=1m/\sec$. Find the rate at which the surface area of the box is changing at the instant when $x=4, y=3$ and $z=2$. (Hint $A=2xy+2xz+2yz$) (4 marks)

b) Given $y=x^3(3x-2)$.

i. Identify the x-coordinates of any local maximum points and local minimum points. (3 marks)

ii. Identify the interval(s) of concave up and interval(s) of concave down. (3 marks)

Continued...

Question 4 (10 marks)

a) Find:

i. $\int \frac{x^3 + 2x^2 - 15x}{x+5} dx$ (2 marks)

ii. $\int (3t^2 - 2t)(5t^3 - 5t^2 + 4)^4 \, dt$ (3 marks)

b) Use integration by parts to evaluate $\int_0^{\pi/2} x^2 \cos x dx$. (5 marks)

Question 5 (10 marks)

a) Find the area of the region bounded by the curves $y = x^2$ and $y = 8 - x^2$. (5 marks)

b) i. Sketch the region bounded by the curve $y = x^2$, the x-axis and the line $x = 4$. (2 marks)

ii. Find the volume of the solid of revolution when the region is rotated about the x–axis. (3 marks)

Question 6 (10 marks)

a) Solve $y' + 3y = e^{-4x}$ given $y(0) = 2$. (4 marks)

b) Solve the second order differential equation $y'' - 4y = 2x + e^{3x}$. (6 marks)

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APPENDIX

A. Differentiation Rules

$\frac{d}{dx}[x^n] = nx^{n-1}$; n is any real number

$\frac{d}{dx}[f(x).g(x)] = f(x)g'(x) + f'(x)g(x)$; The Product Rule

$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$; The Quotient Rule

$\frac{d}{dx}[f(g(x)] = f'(g(x)).g'(x)$; The Chain Rule

$\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1}.g'(x)$; The power rule combined with the chain rule:

$\frac{d}{dx}[\sin x] = \cos x$ $\quad$ $\frac{d}{dx}[\cos x] = -\sin x$ $\quad$ $\frac{d}{dx}[\tan x] = \sec^2 x$

$\frac{d}{dx}[\sec x] = \sec x \tan x$ $\quad$ $\frac{d}{dx}[\cot x] = -\csc^2 x$ $\quad$ $\frac{d}{dx}[\csc x] = -\csc x \cot x$

$\frac{d}{dx}[e^x] = e^x$ $\quad$ $\frac{d}{dx}[\ln x] = \frac{1}{x}; \quad x > 0$

B. Basic Integration Formulas

$\int cf(x)\,dx = c\int f(x)\,dx$ $\quad$ $\int k.dx = kx + C$

$\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$ $\quad$ $\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$

$\int e^x\,dx = e^x + C$ $\quad$ $\int \frac{1}{x}dx = \ln|x| + C$

Integration by-parts: $\int u\,dv = uv - \int v\,du$

Volume (disk) $= \pi\int_a^b (f(x))^2\,dx$ $\quad$ **Area** $= \int_a^b (f(x) - g(x))dx$

Volume (washer) $= \pi\int_a^b \left[(f(x))^2 - (g(x))^2\right]dx$